Use and optimization of the Proper Generalized Decomposition (PGD) to solve complex problems

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Context : Nowadays, numerous scientific problems remain intractable using classical numerical techniques as they involve a huge number of unknowns of parameters. This is for instance the case for quantum mechanics problems, or for structures mechanics problems with uncertain data. This phenomenon is currently denoted “curse of dimensionality”. However, a very promising approach, called Proper Generalized Decomposition (PGD), is intensely studied to circumvent the issue; it consists in a separated variables decomposition of the solution, with modes, of the form (for a space-time problem):

$$u(x,t) = \sum_i \psi_i(x)\lambda_i(t)$$

In many applications, few modes are necessary to obtain a very accurate (and with a reasonable computational cost) numerical solution (cf. Fig1).

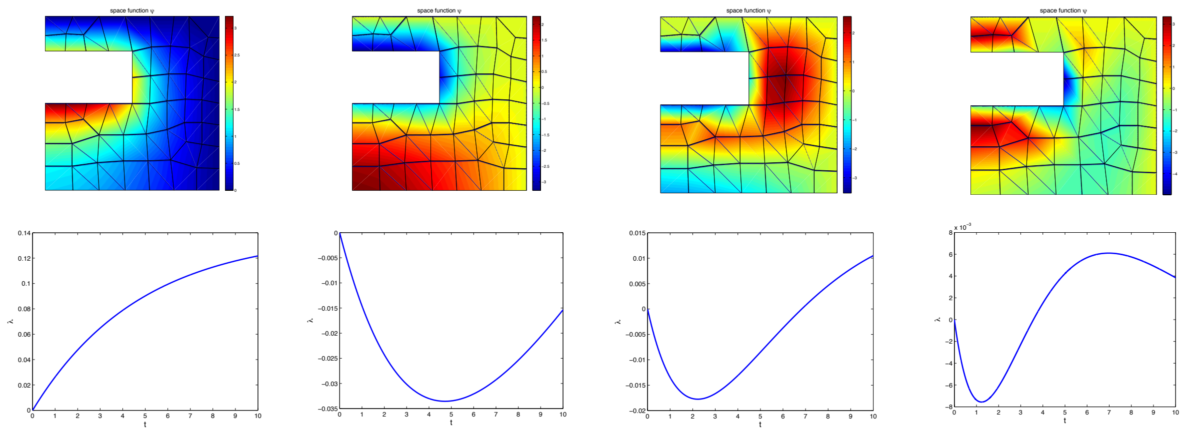

Figure 1 : representation of the first four modes for the solution of a transient thermics problem : space functions $\psi_i(x)$ (upper row), and time functions $\lambda_i(t)$ (lower row).

Objectives : In this work, we aim at optimizing the performances of PGD as regards two criteria. First, we want to set up error indicators that enable to control and adapt the PGD representation (do we need more modes? do we need a better FE mesh to compute theses modes?....). In a second stage, we want to optimize the PGD computation when it is dedicated to the prediction of some quantities of interest instead rather than the global solution itself.

Schedule of the training:

1) Bibliography on PGD and application on simple test cases;
2) Calculation of error indicators to drive the adaptation process;
3) Validation of adaptation performances on numerical examples;
4) Optimization of PGD to predict quantities of interest.